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Intrinsic and Fundamental Decoherence: Issues and Problems

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Abstract. We investigate the meaning of gravity-induced decoherence in quantum theory, known as 'intrinsic' or 'fundamental' decoherence in the literature. We explore a range of issues relevant to this problem, including the meaning of modified uncertainty relations, the interpretations of the Planck scale, the distinction between quantum and stochastic fluctuations and the role of the time variable in quantum mechanics. We examine the specific physical assumptions that enter into different approaches to the subject. In particular, we critique two representative approaches that identify time fluctuations as the origin of intrinsic or fundamental decoherence: one that models the fluctuations by stochastic process and one that purports to derive decoherence from the quantum fluctuations of real clocks.

1. Introduction

1.1. Quantum, Intrinsic and Gravitational Decoherence

We begin by distinguishing between the nature of quantum, intrinsic and gravitational decoherence, as these terms are used by different authors in special contexts for specific purposes. **Quantum decoherence** refers to the loss of coherence in a quantum system for various reasons, commonly due to its interaction with an environment. It could be formulated in different ways, through decoherent or consistent histories [1] or via open-system dynamics [2]. Much work has been done since the beginning of the 90's that we now think we have a better understanding of this issue (see books and reviews, e.g., in [3]).

Intrinsic or fundamental decoherence refers to some intrinsic or fundamental conditions or processes which engender decoherence in varying degrees but universal to all quantum systems. This could come from (or could account for) the uncertainty relation [4, 5], some fundamental imprecision in the measuring devices (starting with clocks and rulers) [6] or in the dynamics [7], or treating time as a statistic variable [8]. Hereby lies the possibility of alternative theories of quantum mechanics, such as theories based on a stochastic Schrodinger equation. This alternative form has been proposed by many authors, notably, Ghirardi, Rimini, Weber and Pearle [9] Diosi[10], Gisin [11], Penrose [12], Persival [13] and Hughston [14] et al who suggest gravity as the origin of the modification, and Adler [15] who views quantum mechanics as emergent from a more fundamental theory. (See [16] for a comparison of the approaches by Milburn, Adler, Diosi and Penrose.) Here, the search is mainly motivated by an uneasiness in the awkward union between general relativity and quantum mechanics, and the context can be in the settings of quantum gravity or at today's low energy with a classical spacetime structure and quantum matter fields. Some protagonists believe that gravity may be needed to make quantum physics more complete, and since universal conditions are involved in these investigations, gravitational decoherence is often brought up for this purpose. For a discussion on issues of decoherence in quantum gravity, see, e.g., [17] and references therein.

Gravitational decoherence refers to the effect of gravity on the decoherence of quantum systems. In principle it also pertains to quantum gravitational effects operative presumably at the Planck scale, but we separate our consideration of these two regimes so the role of gravity will not be confused. We save the term gravitational decoherence to refer to gravity as described by general relativity. For this, even weak gravity is thought to act differently in bringing decoherence to a quantum system than other matter fields. Some topics explored by various authors are listed in [17].

2. Intrinsic Decoherence, Modified Uncertainty Principle and Clock Accuracy

From the low-energy viewpoint, the incorporation of gravity into the quantum description involves a theory in which the fundamental parameters are the Planck constant $\hbar$, the speed of light c, and the Newton's constant G whereby the relevant length-scale would then appear to be the Planck length $l_p = (\hbar G/c^3)^{1/2}$. The usual interpretation is that l_p defines the scale in which quantum effects become important in the description of gravity. A complementary interpretation (relevant to the present discussion) is that l_p defines a divergence from the predictions of ordinary low energy physics, which should arise from the inclusion of gravity effects.

In an alternative line of thought, the Planck length is believed to induce a modification (or limitation) to the familiar concepts of quantum theory. In particular, it is often suggested that l_p places a limit to the measurability of lengths and thus causes a modification of the usual Heisenberg uncertainty relations. A recurrent theme is that these modifications may be implemented by small changes in the standard quantum mechanical formalism (i.e. deformation of the canonical commutation relations, changes in the dynamics), which could lead to effects observable even at low energies.

In the following we have selected two prominent arguments in relation to the Planck-scale-induced effects that suggest a modification of quantum theory. We give a brief summary of their claims followed by some remarks on the central issues.

2.1. Two arguments for modified uncertainty relation

2.1.1. Wigner's argument extended Wigner's original paper [4] on 'fundamental imprecision' of clocks and rulers made no reference to gravity. The argument extended to include gravity we follow here was first proposed by Karolyhazy [5]. This train of thought is recently used by Ng [20] to suggest relations between clocks, black holes and quantum computation (see also [21]).

To measure the distance l between a clock of mass m and a mirror, one sends a light signal from the clock, which is reflected at the mirror and is then received back at the clock. If the clock's position uncertainty is δl, after time $t = 2l/c$, the spread in the clock's uncertainty has increased by a factor of $\hbar t/m\delta l$ due to momentum uncertainty, so that the total spread is $\delta l + \frac{\hbar l}{m\delta l c}$. This has a minimum of

$$\delta l^2 \geq \frac{\hbar l}{mc}, \tag{1}$$

which decreases with mass. Hence, from this argument, heavier clocks are more accurate.

On the other hand, the uncertainty δl is at least as large as the size of the clock, which is constrained to be smaller than the Schwarzschild radius of a body with mass m. Hence,

$$\delta l \geq \frac{Gm}{c^2}. \tag{2}$$

Multiplying these equations one obtains

$$\delta l \geq l_p (l/l_p)^{1/3}. \tag{3}$$

2.1.2. Heisenberg microscope One considers the standard Heisenberg microscope argument for the uncertainty relation, but takes into account the gravitational interaction between a measured particle and an EM field. A photon of energy E will act on the electron with a force of the order $\frac{GE}{c^2L^2}$, where L is the typical size of the cavity. The characteristic time of the interaction is L/c, hence the force will change the electron's position by a factor of

$$\delta x = \frac{1}{m}\frac{GE}{c^2L^2}\left(\frac{L}{c}\right)^2 = \frac{GE}{mc^4}. \tag{4}$$

The photon's momentum is E/c and this equals the uncertainty Δp in the determination of the electron's momentum. We therefore have an uncertainty $\delta x = \frac{G}{mc^3}\Delta p$. Adding this to the inherent quantum uncertainty yields

$$\Delta x \geq \min\{\frac{\hbar}{\Delta p}, \frac{G}{mc^3}\Delta p\} = \min\{\frac{\hbar}{\Delta p}, l_p^2\frac{\Delta p}{\hbar}\} \geq l_p. \tag{5}$$

This argument can be made more precise, using linearized general relativity [22].

2.2. Comments

Note that the two arguments above lead to rather different expressions. The first argument provides a non-trivial bound in how accurately a length can be measured – greater lengths have higher intrinsic uncertainty. The second provides a modification to the uncertainty relation.

Both arguments above have features common to all approaches to this subject matter. The estimates for uncertainty are based on separate arguments that involve different length scales: the atomic scale for the use of the uncertainty principle and the macroscopic scale for the black hole argument. These results are then extrapolated to a regime where they are both assumed to be valid jointly: the Planck length then appears for dimensional reasons. The protagonist would argue that it is the best one can do in order to obtain at least a rough estimate of the consequences of a quantum theory of gravity from the low energy viewpoint. However, such extrapolation involves a leap of faith, our present state of knowledge being limited to low energy phenomena.

We urge the exercise of caution in indiscriminately extending the physics of low energies (compared to the Planck scale) to unknown domains. They usually involve specific *physical* assumptions, which may or may not be valid. We mention two aspects:

2.2.1. Problems in using macroscopic measurement devices to infer microscopic physics Both arguments are developed with reference to the process of quantum measurement. *A priori*, it is not clear that this language can be applied unreservedly to all length scales. Our measurement devices, such as clocks, are typically macroscopic or mesoscopic physical systems, which consist of a large number of microscopic particles. Taken at

face value, the arguments above seem to imply that the notion of measurement devices makes sense at arbitrarily small length scales: in both cases a detection of photons (either scattered by the atoms or absorbed by the clock) is involved. However, the measurement devices that can achieve this task usually consist themselves of atoms and the atomic scale places strong constraints on the accuracy of measurement.

In the first example, the accuracy in measurement of position cannot be finer than the size of the smallest constituents of the device. In particular, the frequency ω of the detected light signal induces an uncertainty $\delta t = 1/\omega$ for the time of detection. However, ω cannot be taken arbitrarily large in known physics: if ω^{-1} is of the order of the plasma frequency of the atomic electrons in the coating, then the mirror is nonreflective and the thought experiments involving such devices lose their meaning.

In the second example, there is a more fundamental limitation placed by relativistic quantum physics. The minimal resolution for the measurement of position by photons is the de Broglie wavelength of the electron $\hbar/m_e c$, which corresponds to the energy scale $2m_e c^2$ of pair creation for the lightest particles interacting with the electromagnetic field. At such energies, the notion of the Heisenberg microsope becomes problematic and a quantum field theoretic treatment of the problem is mandatory.

2.2.2. The Risk of violating stringent consistency conditions in Planck scale physics
The second example also involves a comparison of the gravity-induced and of the familiar quantum uncertainty. While at the present state of knowledge these sources of uncertainty are independent, this independence is not guaranteed to hold at the Planck scale where quantum and gravitational theories are expected to be unified or interlinked. Even with some degree of linkage, certain self-consistency requirement ought to exist and be respected which may work against simple extrapolations from low energy phenomenology. In fact, there is at least one example, in which the justification of the uncertainty principle employs arguments from gravity (Bohr's refutation of Einstein's box experiment)[23]: in this case the gravity uncertainties are employed in order to justify the quantum uncertainty relation.

In essence, most proposals of this nature starting from low energy physics and extrapolating their predictions to Planck scale regime involve some selective processes. They focus on some favored (usually the easy) aspect but leave out the potential limitations from other known physics which could exert equal or more stringent bounds on the validity of these arguments. The reasoning which follows from them is usually incomplete or inconsistent, which makes the interpretation (usually grounded on measurements at low energies) problematic.

2.3. Uncertainty and noise

We argued before that the derivation of gravity-modified uncertainty in terms of measurement inaccuracies is not straightforward, as it extrapolates results of quantum measurement theory at low energies into a new regime, where habitual concepts and

ordinary tools may fail to apply. We note that the most concrete understanding of quantum uncertainties is as mean deviations in the statistics of measurements. Any other viewpoint (for instance, thinking of uncertainties as referring to the definability of the physical magnitudes involved) is more elusive and it depends on one's preferred interpretation of quantum mechanics.

But even if we separate the quantum uncertainties from a measurement process, the existence of such uncertainties does not say anything about the physics they encode. In particular, the statistical theory one employs for its description is a modal simplification. By this we mean, that we have no *a priori* justification in modeling these uncertainties by stochastic processes which are intrinsically classical. The stochastic assumption may only be valid for uncertainties in measurements *classical* in nature i.e., containing randomness at the macroscopic scale. In contrast, quantum uncertainties are different in nature – they cannot be modeled by a stochastic measure as they involve non-localities and correlations with no analogues in classical probability. See [17] for some discussions about the difference between quantum and stochastic effects pertaining to decoherence.

2.4. Limitations in using particle as a field probe

In the above discussions we urge a closer scrutiny on the physical assumptions involved in any interpretation of new Planck scale effects from the low energy viewpoint. Clearly, there *are* physical effects associated with the introduction of the Planck scale, but it is not straightforward to infer how they will be manifested in the low-energy regime. In particular, one should be a little wary whether a classical stochastic description of uncertainties does justice to the full quantum features of the variables involved.

To see this we consider an analogous situation related to the measurement of the quantum electromagnetic fields by test probes of mass m and charge q. The classic analysis of Bohr and Rosenfeld [24] demonstrates that there is an uncertainty ΔF in the determination of the electromagnetic field (EMF) strength F given by

$$\Delta F l^3 \geq \frac{q\hbar}{mc}, \tag{6}$$

where l is the typical length scale of the interaction region and the following conditions must also hold

$$lmc \geq \hbar \qquad\qquad lm \geq q^2c^2. \tag{7}$$

The quantity $\Delta F l q$ is a measure of the uncertainty in the electromagnetic energy ϵ_{el} of the particle. Since the particle description is inadequate if $\epsilon_{el} > mc^2$, we obtain the condition

$$l \geq \left(\frac{\hbar}{c^3}\right)^{1/2} \frac{q}{m}. \tag{8}$$

This sets a scale to the size of the domain in which the particle can identify a classical configuration for the EMF. For smaller regions, the interaction between particle and field is fully quantum and it is impossible to make a sharp distinction between them.

In gravity the role of F is played by the connection Γ and of $\sqrt{\hbar/c^3}q$ by $l_p m$. Hence, the limit (8) becomes $l > l_p$. Transferring the reasoning from the electromagnetic case, one would say that l_p determines the scale in which the coupling of the particle to the gravitational field must take into account the full quantum nature of the latter. In this reasoning, it does not imply anything about the structure of spacetime or of a necessary modification in the dynamics of low energy, but is a statement of the fact that at l_p a particle fails to detect the classical aspects of the gravitational field's character.

The point we want to make is that the limitations posed by the Planck length are not *a priori* different from those placed by the scale $\sqrt{\hbar/c^3}q/m$ in quantum electrodynamics: At this scale quantum field effects are strong. In electrodynamics the fluctuations from these effects are fully quantum. Any effect they cause at low energy is also inherently quantum: there is no justification in treating them like classical fluctuations (i.e., described by a stochastic process), unless we assume a specific regime for the quantum field. In particular, the effects of the fluctuations of the EMF at low energies ($E << mc^2$) have been studied in [25, 26, 27, 28, 29]. There, it was shown that the 'noise' induced by these fluctuations is non-Markovian and does not cause significant decoherence effects in the microscopic regime. In other words, the coherence of the EMF vacuum does not allow for the *a priori* generation of classical (i.e., decohering) fluctuations in the quantum motion of the particle.

3. Intrinsic Decoherence from Time Fluctuations

3.1. Time fluctuations

A key theme in the approaches to intrinsic decoherence is that it is induced by the fluctuations or uncertainties in time caused by gravity. This idea originates with Penrose [12], who noted that a superposition of two states with different mass density distributions would lead to a superposition of two different spacetimes, and hence an ambiguity in the notion of the time variable, through which the superimposed quantum states evolved. He then suggested that this ambiguity might imply that such superpositions are not stable and thus decay. This would suggest that (in the non-relativistic regime) in effect there is decoherence in the 'basis' of energy density. A special case, if the superimposed states have support in the same region of space, the decoherence would be in the energy basis.

The argument above is stated using only the basic principles of quantum theory and general relativity. Without further assumptions it only suggests the plausibility of an intrinsic decoherence effect and an order of magnitude estimate in the non-relativistic regime. However, if one makes additional assumptions about the nature of the time fluctuations, it is possible to derive a master equation for the quantum state and thereby study decoherence as a dynamical effect.

The additional assumptions usually refer to the statistical properties of the time fluctuations. For example, Diosi has considered that the fluctuations of time are

expressed in terms of fluctuations of the Newtonian potential, which can be seen as a random field satisfying a Markovian process [10].

3.2. Milburn's Intrinsic Decoherence

We next examine one representative scheme of intrinsic decoherence, that proposed by Milburn [7]. Actually, Milburn's discussion is in a context more general than that of gravitational induced fluctuations. He studies the effects of randomness in the preparation of experiments performed on relativistic systems. He assumes that there is a (perhaps fundamental) minimal fluctuation ϵ in the determination of some physical quantity A. Let $\hat{g}$ be the generator corresponding to translations in the value of A (e.g., the Hamiltonian if A is time, the momentum if A is a spatial displacement). Assuming that the fluctuations are described by classical probability theory, there is a probability $p_n(\epsilon, \theta)$ that n random shifts of order ϵ will result in a change of the parameter A from $A = 0$ to $A = \theta$. Then one can write the following equation for the density matrix

$$\rho_\epsilon(\theta) = \sum_{n=0}^{\infty} p_n(\epsilon, \theta) e^{-in\epsilon\hat{g}} \rho(0) e^{in\epsilon\hat{g}}, \tag{9}$$

with the condition that for $\epsilon = 0$ this law corresponds to a unitary transformation

$$\lim_{\epsilon\to 0} \rho_\epsilon(\theta) = e^{-i\theta\hat{g}} \rho(0) e^{i\theta\hat{g}}. \tag{10}$$

The equation above defines a positive semigroup [18, 19]. There is substantial freedom in its choice, as it depends on the determination of the probability function p_n. A convenient choice is the Poisson distribution

$$p_n(\epsilon, \theta) = \frac{(\theta/\epsilon)^n}{n!} e^{-\theta/\epsilon}, \tag{11}$$

which leads to a semigroup equation

$$\frac{d\rho}{d\theta} = \frac{1}{\epsilon}\left(e^{-i\epsilon\hat{g}} \rho e^{i\epsilon\hat{g}} - \rho\right). \tag{12}$$

Milburn then explores this semi-group transformation for the special case of time-translations ($\hat{g} = \hat{H}$) and at displacements ($\hat{g} = \hat{P}_i$), in some spatial direction labeled by i. This choice allows for the preservation of energy and momentum, He is mainly interested in uncertainty relations. In the first case (and assuming a specific scheme for time measurements) he finds an exponential degradation of the accuracy: $\delta t \sim e^{at}$, in terms of some state-dependent factor a.

In the second case, he finds (for an initial Gaussian state)

$$(\delta X)^2 (\Delta p)_0^2 \geq \frac{1}{4} + \epsilon X (\Delta p)_0^2, \tag{13}$$

where X is the particle's distance from the origin, δX the position uncertainty and $(\Delta p)_0$ is the momentum uncertainty of the initial state. He then compares this result to the analogous expressions for the quantum-gravity-modified uncertainty relations, by identifying ϵ with the Planck length. This construction is fairly general and it can be generalized to the relativistic regime.

3.3. Comments

Strictly speaking, Milburn's formalism deals with the effect of incremental random effects on a quantum system. A priori, there is no compelling reason why the Planck scale, or gravity needs to appear. To make the connection, one identifies the minimal shifts ϵ with the fundamental uncertainties in measurement of position or time arising from the Planck regime. However, in doing so one is making a very strong assumption, namely, that these fluctuations are stochastic and they can thus be modeled by a probability distribution $p_n(\epsilon, \theta)$.

However, quantum effects cannot in general be described in terms of stochastic processes. Coherence and nonlocality (e.g., Bell's theorem) are inherent quantum properties which are lost in a stochastic description. Quantum theory involves 'interference' effects which do not allow the definition of a stochastic measure. Milburn' s assumption inconspicuously treats the Planck-scale fluctuations, what he calls the 'quantum' foam, like fluctuations observed in the macroscopic world. It also ignores possible quantum correlations between the quantum system and its environment which is fundamentally also quantum in nature. Only after coarse-graining (often rather drastically) could it begin to show some classical stochastic features. Interchanging quantum fluctuations with classical noise loses precious quantum information which changes the character of the quantum system fundamentally.

This is particularly pertinent with fluctuations that refer to time. Time occupies a special position in quantum theory: there is no natural time operator in quantum mechanics, and the construction of probabilities for time is rather intricate as even elementary problems like the time-of-arrival indicate. The notion of *quantum* temporal fluctuations is therefore both technically and conceptually different from fluctuations of usual observables (see [30] and references therein, also [31]).

Concerning possible modifications in the uncertainty relation, we note that such modifications are typically generated by non-unitary dynamics. For example, they have been studied in models like quantum Brownian motion [32, 33]. The growth of uncertainties due to noisy-dynamics is a well-established effect. It follows from the fact that the density matrix of a pure state becomes mixed. The uncertainties obtained from such treatments are not fundamentally quantum, in the sense that their origin does not lie in the limits posed by the non-commutativity of operators, but in the *degradation of predictability* induced by the external noise.

We note that the critique above refers to any approach towards gravitationally induced decoherence that makes specific assumptions about the statistical behavior of the temporal fluctuations. However, in Penrose's proposal [12] such assumptions are not made (at the price of not writing a master equation) but the prediction of gravitationally induced decoherence proceeds mainly from order-of-magnitude estimations and the structural incompatibility between quantum mechanics and general relativity in the treatment of time.

4. Fundamental Decoherence, Quantum Mechanics with Real Clocks

Another proposal worthy of some closer scrutiny is the so called fundamental decoherence from quantum gravity of Gambini, Porto and Pullin (GPP) [6]. A distinguishing feature in this approach is that it purports to describe a general scheme for the fully *quantum* treatment of temporal fluctuations. Their argument goes through the following steps:

4.1. GPP's Theses

(i) Unlike in standard quantum mechanics (QM) where the spacetime structure is fixed, in quantum gravity one cannot rely on a time t external to the system. For time evolution to make sense, it is necessary that one introduces variables in the system's phase space that take the role of clocks and rulers. One then has to express the evolution law in terms of readings for these clocks and rulers (relational time).
(ii) Clocks are also quantum systems, hence they are also subject to quantum fluctuations. Expressing time evolution in terms of measurable time leads to non-unitary dynamics, and therefore decoherence.
(iii) One can translate the usual Schrödinger evolution of standard QM into a form, in which the temporal parameter T is the reading of the clock. Since T is a quantum variable, it includes some randomness and is not related to t through a delta function. Following a specific procedure, one can then obtain a master equation for the density matrix, which is of the Lindblad type. Hence the use of a non-ideal clock may generate decoherence, which GPP call 'fundamental decoherence'. The relevant diffusion parameter is determined by the intrinsic uncertainty of the clock variable.
(iv) At the Planck scale, Wigner's argument extended by Karolyhazy and Ng (i.e. black hole creation if we try to measure space or time with arbitrary precision) can be invoked to argue that there is a lower limit to the possible resolution of clocks. Hence, the arguments above apply and there has to be fundamental decoherence.

4.2. Comments

Assumption (i) involves specific interpretations of time in quantum gravity – see [34] and [35] for critique. This is not the only possible interpretation; it is however reasonable and widely held. However, even accepting (i), step (ii) does not necessarily follow. The reason is that there is no *a priori* guarantee that the effective dynamics obtained from writing the evolution in terms of clock time will lead to decoherent dynamics. A quantum clock may very well exhibit strong quantum coherence properties, which will be inherited to the evolution of the system. In other words, it is necessary that the fluctuations of the clock are sufficiently classical so that the effective dynamics will be decoherent.

Hence, to support (ii) one has to demonstrate explicitly a case that dynamics written in terms of clock time leads to decoherence. GPP have provided an argument

for this purpose [6]. Note that while the argument was initially phrased in the context of quantum gravity with discrete structure, it also holds for continuous time and has an analogue within the context of ordinary non-relativistic quantum mechanics. The key point in GPP's argument is a specific use of quantum probability, as explained below.

4.3. Key Point

In standard QM, the time t is an external parameter of the system (in discrete gravity this role is played by the time-step n). Let T be the clock variable. One can then introduce projectors along the range of T, say P_{Δ_T}, which correspond to T taking values in an interval Δ_T.

Similarly one introduces projectors $P_{\Delta_O}(t)$ corresponding to an observable O taking values in a set Δ_O. The key step in GPP's assumption is the following 'expression' for the conditional probability that "the observable O lies in Δ_O provided T lies in ΔT."

$$\mathcal{P}[O \in \Delta_O | T \in \Delta_T] = \lim_{\tau\to\infty} \frac{\int_0^\tau dt Tr[P_{\Delta_O}(t) P_{\Delta_T}(t) \rho_0 P_{\Delta_T}(t)]}{\int_0^\tau dt Tr[\rho_0 P_{\Delta_T}(t)]}. \tag{14}$$

where $P(t) = U^\dagger(t) P U(t)$ is the Heisenberg time evolution of the projector P; $U(t) = e^{-iHt}$.

From this expression, using arguments of standard probability theory it is easy to change variables in the density matrix $\rho(t)$ (evolved unitarily) and write it in terms of T. Indeed, the spread of T with respect to t will be sufficient to show decoherence. In our opinion, the main issue is how Eq. (14) can be justified in the context of standard quantum theory.

4.4. Critique

We find two problems with this line of reasoning. First, the interpretation of the expression (14) as the conditional probability that "the observable O lies in Δ_O provided T lies in ΔT" is not justifiable by the rules of quantum theory and the standard calculus of probabilities. Second, even with a justifiable expression for such probabilities, propositions such as the above are in fact propositions about histories and are subject to the problems of defining probabilities for quantum histories [1].

4.4.1. On the conditional probability (14) If A and B are two events, the conditional probability of "A provided B" is given by the ratio of the joint probability for A and B to the probability for B, provided the latter is non-zero. Then Eq. (14) defines a conditional probability as above if the numerator in (14) is proportional to the joint probability of O lying in Δ_O and T lying in Δ_T, and the denominator proportional to the probability that T lies in Δ_T at some time t. The proportionality factor must have dimension of time inverse; the usual choice is $C(\tau) = \frac{1}{\tau}$.

Hence, the interpretation of (14) as a conditional probability requires that the joint

probability of O lying in Δ_O and T lying in Δ_T be given by

$$\lim_{\tau\to\infty} C(\tau)\int_0^\tau dt Tr[P_{\Delta_O}(t)P_{\Delta_T}(t)\rho_0 P_{\Delta_T}(t)]. \tag{15}$$

An analogous expression should hold when the time is taken discrete, namely,

$$\lim_{N\to\infty} C(N)\sum_{i=1}^{N} Tr[P_{\Delta_O}(t_i)P_{\Delta_T}(t_i)\rho_0 P_{\Delta_T}(t_i)]. \tag{16}$$

The question is how the probabilistic interpretation of the expressions above are justified in the context of standard quantum probability. In Eq. (16), the term $p_i := Tr[P_{\Delta_O}(t_i)P_{\Delta_T}(t_i)\rho_0 P_{\Delta_T}(t_i)]$ can be interpreted as the probability that $O \in \Delta_O$ at time t_i and $T \in \Delta_T$ at time t_i‡. Now, GPP consider the probability (16) as corresponding to the statement that "$O \in \Delta_O, T \in \Delta_T$ at some time t_i". By the expression "at some time" one means "either at time t_1, or at t_2, or at t_3 etc for all times t_i." Hence, they seem to argue, it should equal the sum of the probabilities that "$O \in \Delta_O, T \in \Delta_T$ at time t_i".

In effect, this definition relies on the following assumption

"probability of $O \in \Delta_O, T \in \Delta_T$ at time t" + "probability of $O \in \Delta_O, T \in \Delta_T$ at time t'" = "probability of $O \in \Delta_O, T \in \Delta_T$ either at time t or at time t'".

However, this assumption is erroneous. The reason is that the additivity of probabilities holds only for *exclusive* alternatives, i.e. if their join is the empty set§. This is not the case here. To see this, one should note that in the space of alternatives for the system, t_i is *not* a random variable, but a parameter of the system's histories. This means that, unlike what is the case for random variables, alternatives labeled by different values of t_i are *not necessarily* exclusive‖. This follows from the 'logical' structure of alternatives involving time in quantum theory–see [36, 37] ofr details. The alternatives considered here fall into the class of the so-called spacetime coarse-grainings [38, 31].

An example from standard probability theory serves to illustrate the point above. Let us consider the space of alternatives for the position x of a particle at different times. The proposition "the particle lies at $x = x_1$ at time t_1" is not disjoint to the proposition that "the particle lies at $x = x_1$ at time t_2": clearly, there are paths that satisfy *both* propositions, for example a path that satisfies $x = x_1$ at all times t_i. One cannot therefore obtain the probability of their join by summing the individual probabilities.

Eq. (16) can be viewed as a time average of single-time probabilities for $O \in \Delta_O, T \in \Delta_T$, but as we showed this time-average is not in any sense related to the

‡ If O and T do not commute, then there is the tacit assumption that the determination of T takes place just prior to the determination of O, i.e. T is determined at $t_i - \epsilon$ and O at time $t_i + \epsilon$, where ϵ is very small and is eventually taken to zero.

§ We remind the reader that for two sets A and B, $p(A \cup B) = p(A) + p(B) - p(A \cap B)$.

‖ Note that the above holds for any parameter, discrete or continuous, that can be used as a label for the causal ordering of the properties of the system. It does have to be named 'time' or to coincide with the time of the non-relativistic theory.

proposition that "$O \in \Delta_O, T \in \Delta_T$ at any time t_i". In fact it is not clear that it refers to any physical statement at all.

To construct the correct expression for the probability that "$O \in \Delta_O, T \in \Delta_T$ at some time t_i" one notes that this proposition is the negation of the proposition that "$O \notin \Delta_O, T \notin \Delta_T$ at *all* times t_i". Hence, the probabilities for these two alternatives add up to unity. Now, the probability for "$O \notin \Delta_O, T \notin \Delta_T$ at *all* times t_i" is given by the standard expression

$$\begin{aligned} &Tr\left[(1-P_{\Delta_O})(t_N)(1-P_{\Delta_T})(t_N)\ldots(1-P_{\Delta_O})(t_1)(1-P_{\Delta_T})(t_1)\rho_0\right. \\ &\left.\times(1-P_{\Delta_O})(t_1)(1-P_{\Delta_T})(t_1)\ldots(1-P_{\Delta_O})(t_N)(1-P_{\Delta_T})(t_N)\right], \end{aligned} \tag{17}$$

where the limit $N \to \infty$ may be taken at the end. Then the probability for the proposition "$O \in \Delta_O, T \in \Delta_T$ at some time t_i" equals

$$\begin{aligned} &p(O \in \Delta_O, T \in \Delta_T) = \\ &1 - \lim_{N\to\infty} Tr\left[\left(\prod_{i=1}^{N}(1-P_{\Delta_O})(t_i)(1-P_{\Delta_T})(t_i)\right)^{\dagger}\rho_0\prod_{i=1}^{N}(1-P_{\Delta_O})(t_i)(1-P_{\Delta_T})(t_i)\right], \end{aligned} \tag{18}$$

which clearly is very different from (16).

4.4.2. The issue of defining probabilities for histories Eq. (18) is obtained by standard probabilistic arguments together with the quantum reduction rule. However, it has problems typical to all probability assignment for histories, i.e. propositions about a quantum system that refer to more than one instant of time [1]. The presumed probabilities (18) – but also the probabilities (16) – do not satisfy the Kolmogorov additivity conditions

$$p(O \in \Delta_O, T \in \Delta_T^1 \cup \Delta_T^2) = p(O \in \Delta_O, T \in \Delta_T^1) + p(O \in \Delta_O, T \in \Delta_T^2), \tag{19}$$

for any disjoint sets Δ_T^1, Δ_T^2. This implies that Eq. (18) fails to define a genuine probability measure.

The demonstration is standard [1] and can be seen by direct inspection of Eq. (18).Π

To define a genuine probability measure for the joint values of O and T out of Eq. (18), or (16), one must either restrict to specific sets of histories that satisfy a decoherence condition as in the consistent/decoherent histories approach, or to modify (18) in a way that allows for the definition of a probability measure in the context of measurements – see [39] for details. The former case effectively assumes the decoherence one is supposed to find, while the latter refers to an effectively open system

Π For Eq. (16), the probability corresponding to $O \in \Delta_O, T \in \Delta_T^1 \cup \Delta_T^2$ can easily be shown to differ from the sum of the probability for $O \in \Delta_O, T \in \Delta_T^1$ to the probability for $O \in \Delta_O, T \in \Delta_T^2$ by an 'interference' term $2Re\left[P_{\Delta_O}(t_i)P_{\Delta_T^1}(t_i)\rho_0 P_{\Delta_T^2}(t_i)\right]$, which is generically non-zero. Hence, Eq. (14) also does not define a genuine probability measure for the space of values for O and T.

[+]. Either way, even with a meaningful construction of conditional probabilities, the domain of validity for the description of evolution in terms of relational time is reduced substantially.

This problem is a special case of a more general fact. If we want to treat time fluctuations within standard quantum theory, we must effectively work within a histories scheme, because a theory with time fluctuations necessarily involves properties of the system at more than one 'moments of time', i.e., histories. One then needs to address the problem of adequately defining probabilities for such histories. This is a highly non-trivial task that adds a substantial degree of difficulty in the problem.

5. Summary

There is substantial motivation for the study of intrinsic or fundamental decoherence in quantum mechanics, especially in relation to gravitational effects.

a. A modification of quantum mechanics at this level could provide a resolution of long-standing problems, such as the quantum measurement problem.
b. The incompatibility between quantum theory and general relativity (especially in the treatment of time) suggests a modification of known physics in the regime in which both theories apply and this could have observable consequences even in non-relativistic physics.
c. It is conceivable that such modifications uncover remnants of processes fully present at much higher energy scales, such as the Planck length.

The aim of such studies is to identify new physics pertaining to the interplay between gravity and quantum theory in a different (and observationally more accessible) regime than the one usually attributed to quantum gravity. We point out any such theoretical treatment may risk making strong *ad hoc* physical assumptions about the nature and origin of such effects, such as intrinsic or fundamental decoherence. One ought to be wary of the danger that predictions drawn from such deductions only reflect the assumptions entering in their derivations.

Here we focussed on the analysis of the basic assumptions implicit in such schemes. We first discussed the notion of a gravity modified uncertainty relations: the Planck scale enters explicitly into such relations. However, this is primarily due to dimensional reasons: derivations of such modifications involve a huge extrapolation of known physics to an unknown regime. Moreover, the existence of fluctuations (say, for time) says nothing about the physics they encode. Emphatically, whether such fluctuations lead to fundamental decoherence depends on the probabilistic theory that describes them: we

[+] Since the expression effectively involves an integral over all times, one should assume a *continuous-time measurement* scheme, i.e. the presence of a device measuring (approximately) both T and O at all times t. In other words, one would have to assume a continuous 'reduction of the wave packet', in order to suppress the generic interference terms in the probability with respect to time.

point out that fluctuations arising from a state that preserves quantum coherence (like for example the vacuum of the electromagnetic field) are not likely to give significant decoherence effects. The invocation of a stochastic process modelling such fluctuations involves *a strong physical requirement* that the relevant quantum degrees of freedom have been already classicalized, or that they are classical to begin with.

When the fluctuations refer to time the situation becomes more complex. Setting aside any conceptual issues about the meaning of temporal fluctuations, their treatment is very intricate at the quantum level, because of the distinguishing role time plays in quantum mechanics and the fact that the corresponding probabilities are in general not well-defined.

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